National
Qualifications
2022 MODIFIED

# X847/77/11

# Mathematics
# Paper 1 (Non-calculator)

FRIDAY, 6 MAY

9:00 AM – 10:00 AM

**Total marks — 35**

Attempt ALL questions.

**You must NOT use a calculator.**

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

## FORMULAE LIST

| Standard derivatives | |
|---|---|
| $f(x)$ | $f'(x)$ |
| $\sin^{-1} x$ | $\dfrac{1}{\sqrt{1-x^2}}$ |
| $\cos^{-1} x$ | $-\dfrac{1}{\sqrt{1-x^2}}$ |
| $\tan^{-1} x$ | $\dfrac{1}{1+x^2}$ |
| $\tan x$ | $\sec^2 x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\ln x$ | $\dfrac{1}{x}$ |
| $e^x$ | $e^x$ |

| Standard integrals | |
|---|---|
| $f(x)$ | $\int f(x)\,dx$ |
| $\sec^2(ax)$ | $\dfrac{1}{a}\tan(ax)+c$ |
| $\dfrac{1}{\sqrt{a^2-x^2}}$ | $\sin^{-1}\left(\dfrac{x}{a}\right)+c$ |
| $\dfrac{1}{a^2+x^2}$ | $\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)+c$ |
| $\dfrac{1}{x}$ | $\ln\lvert x\rvert+c$ |
| $e^{ax}$ | $\dfrac{1}{a}e^{ax}+c$ |

**Summations**

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a\left(1-r^n\right)}{1-r},\ r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

**Binomial theorem**

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

**Maclaurin expansion**

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

## FORMULAE LIST (continued)

**De Moivre's theorem**

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

**Vector product**

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

**Matrix transformation**

Anti-clockwise rotation through an angle, $\theta$, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

**[Turn over**

MARKS

**Total marks — 35**

**Attempt ALL questions**

**1.** (a) Given $y = \dfrac{1-3x}{x^2+4}$, find $\dfrac{dy}{dx}$. Simplify your answer. **3**

(b) Given $f(x) = \operatorname{cosec} 5x$, find $f'(x)$. **2**

**2.** Use Gaussian elimination to solve the following system of equations:

$$x - 2y + z = 4$$
$$2x + y - 3z = 3$$
$$x - 7y - 4z = 9$$

**4**

**3.** Given that $z_1 = 5 + 3i$ and $z_2 = 6 + 2i$, express $z_1\overline{z}_2$ in the form $a + ib$ where $a$ and $b$ are real numbers. **2**

**4.** A curve is defined by the equation $y^3 + 4y = 2xy + 1$.

(a) Use implicit differentiation to find an expression for $\dfrac{dy}{dx}$. **3**

(b) Find the gradient of the tangent to the curve when $y = -1$. **1**

(c) Show that the curve has no stationary point. **2**

**5.** (a) Find, and simplify, the Maclaurin expansion for $e^{-4x}$, up to and including the term in $x^3$. **2**

(b) Hence find the first four terms of the Maclaurin expansion of $\dfrac{3+2x}{e^{4x}}$. **2**

MARKS

**6.** (a) Consider the statement:

For all odd numbers $n$, $n^2 + 4$ is prime.

Find a counterexample to show that the statement is false. **1**

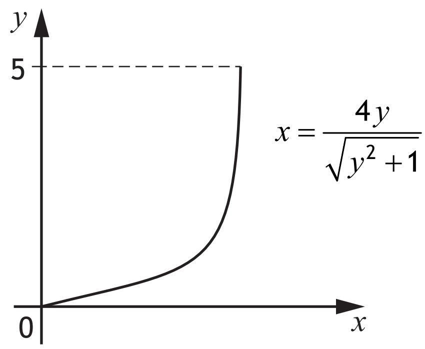
(b) Prove directly that the difference between the cubes of any two consecutive integers is not divisible by 3. **3**

**7.** (a) Use the substitution $u = y^2 + 1$, or otherwise, to find the exact value of

$$\int_0^5 \frac{4y}{\sqrt{y^2+1}}\,dy.$$ **4**

Student engineers are using a 3D printer to make a model.

Relative to a suitable set of axes, the cross-section of the model is **symmetrical about the $y$-axis** and is represented **in the first quadrant** by the curve with equation $x = \dfrac{4y}{\sqrt{y^2+1}}$, $0 \le y \le 5$, as shown in the diagram.




(b) State the area of the cross-section. **1**

(c) Express $\dfrac{y^2}{y^2+1}$ in the form $a + \dfrac{b}{y^2+1}$ where $a$ and $b$ are real numbers. **1**

The curve $x = \dfrac{4y}{\sqrt{y^2+1}}$, $0 \le y \le 5$, will be rotated through $2\pi$ radians about the $y$-axis to make the model.

(d) Find the volume of the model. **4**

**[END OF QUESTION PAPER]**

**[BLANK PAGE]**

**DO NOT WRITE ON THIS PAGE**

[BLANK PAGE]

DO NOT WRITE ON THIS PAGE

[BLANK PAGE]

DO NOT WRITE ON THIS PAGE